

Q(2)

d] Density of states $\rho_{c,v}(E)$ for 2D structure
(Quantum well)

Sec(4)

Assume $L_x = L_y = 1\text{mm}$, $L_z = 10\text{\AA}$

$$k_x = \frac{m\pi}{L_x}$$

$$m = 1, 2, 3, \dots$$

$$k_y = \frac{p\pi}{L_y}$$

$$p = 1, 2, 3, \dots$$

$$k_z = \frac{q\pi}{L_z}$$

$$q = 1, 2, 3, \dots$$

If $L_z = 10\text{\AA}$

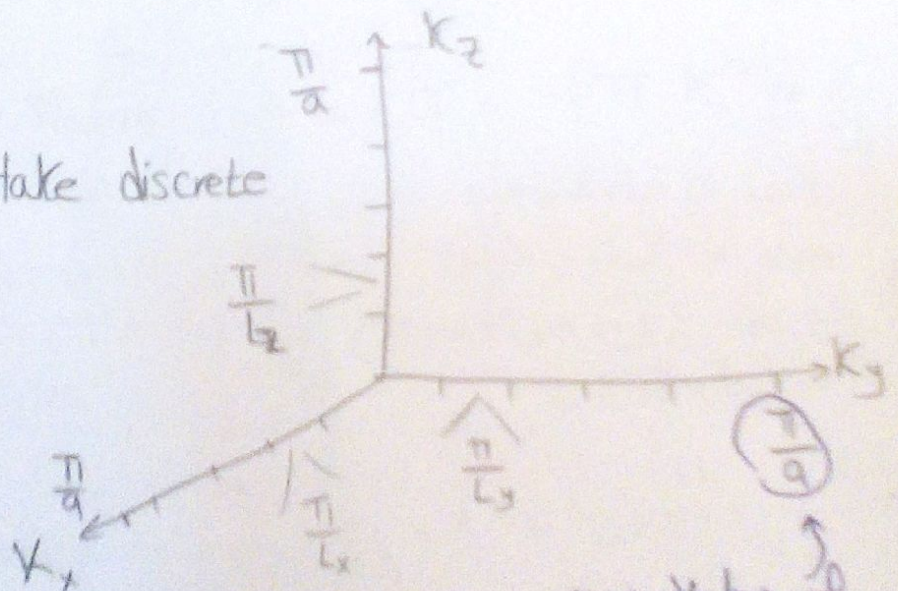
Then k_z will take discrete values

For example:

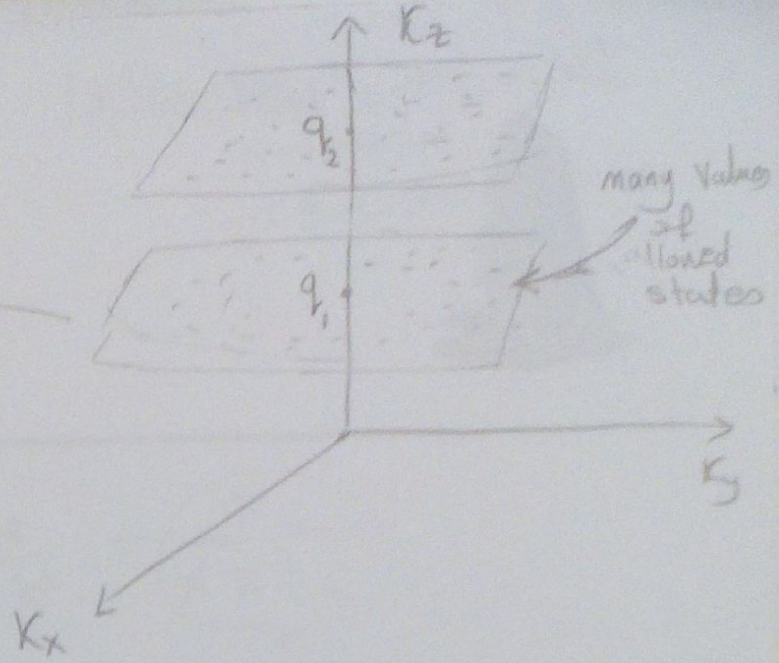
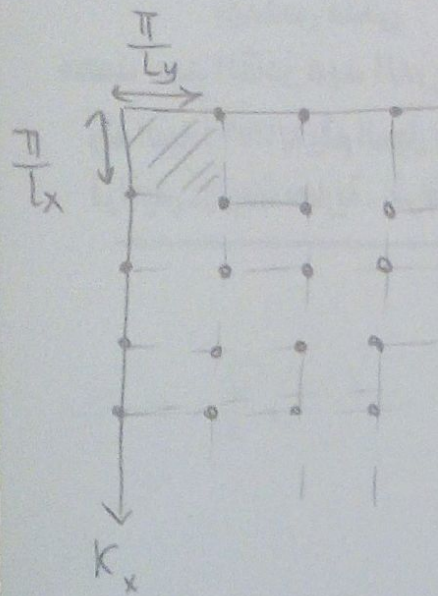
$$k_z|_{q=1} = \frac{\pi}{L_z} = \frac{\pi}{10}$$

$$k_z|_{q=2} = \frac{2\pi}{10} = \frac{\pi}{5}$$

$a = 5\text{\AA}$
in Semiconductor



max. Value of k in 1st Brillouin Zone



Area of one state = $(\frac{\pi}{L_x}) (\frac{\pi}{L_y})$

Area of all allowed states (up to E_n) = $(\pi K^2) \cdot \frac{1}{4}$
 لك قيمه K حاله قيمه E (أخذ الاتجاه الموجب)

Vector in 2D
 دائرة بيضاوية دائرة
 مركزها $(0,0)$

نقطه من الماور $(+K)$
 لانت اقله $(-K)$ هو اتجاه رجوع
 Spin of electron

No. of Allowed states = $\frac{\pi K^2}{4 (\pi^2)} \cdot (L_x L_y) = 2 \times \frac{K^2 L_x L_y}{4 \pi}$

N per unit volume = $\frac{K^2 L_x L_y}{4 \pi (L_x L_y L_z)} = \frac{K^2}{2 \pi L_z}$

$f(E) = \frac{dN}{dE} = \frac{dN}{dK} \cdot \frac{dK}{dE}$

$N = \frac{K^2}{4 \pi L_z} \Rightarrow \frac{dN}{dK} = \frac{2K}{2 \pi L_z} = \frac{K}{\pi L_z}$

CB]

$$E = E_c + \frac{\hbar^2 k^2}{2m_c}$$

$$|k^2| = \cancel{k_x^2 + k_y^2 + k_z^2} = \cancel{k_x^2 + k_y^2} + k_z^2$$

$$\frac{dE}{dk} = \frac{2k\hbar^2}{2m_c} = \frac{k\hbar^2}{m_c}$$

$$\hookrightarrow f(E) = \frac{k}{\pi L_z} \cdot \frac{m_c}{k\hbar^2} = \frac{m_c}{\hbar^2 \pi L_z}$$

Constant does not depend on k

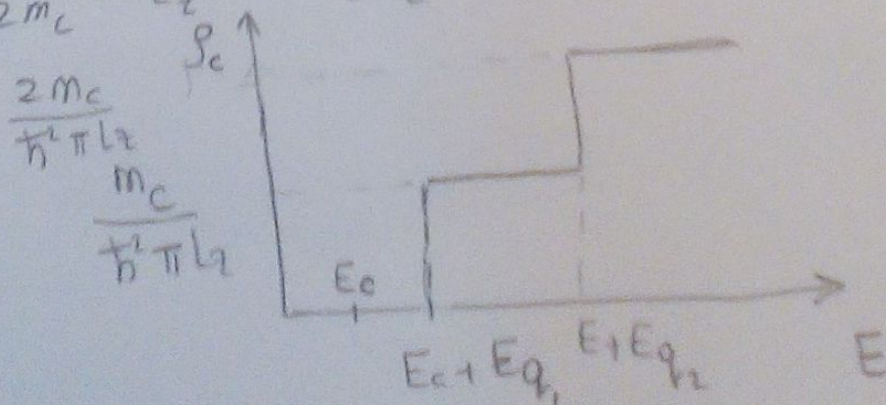
\hookrightarrow

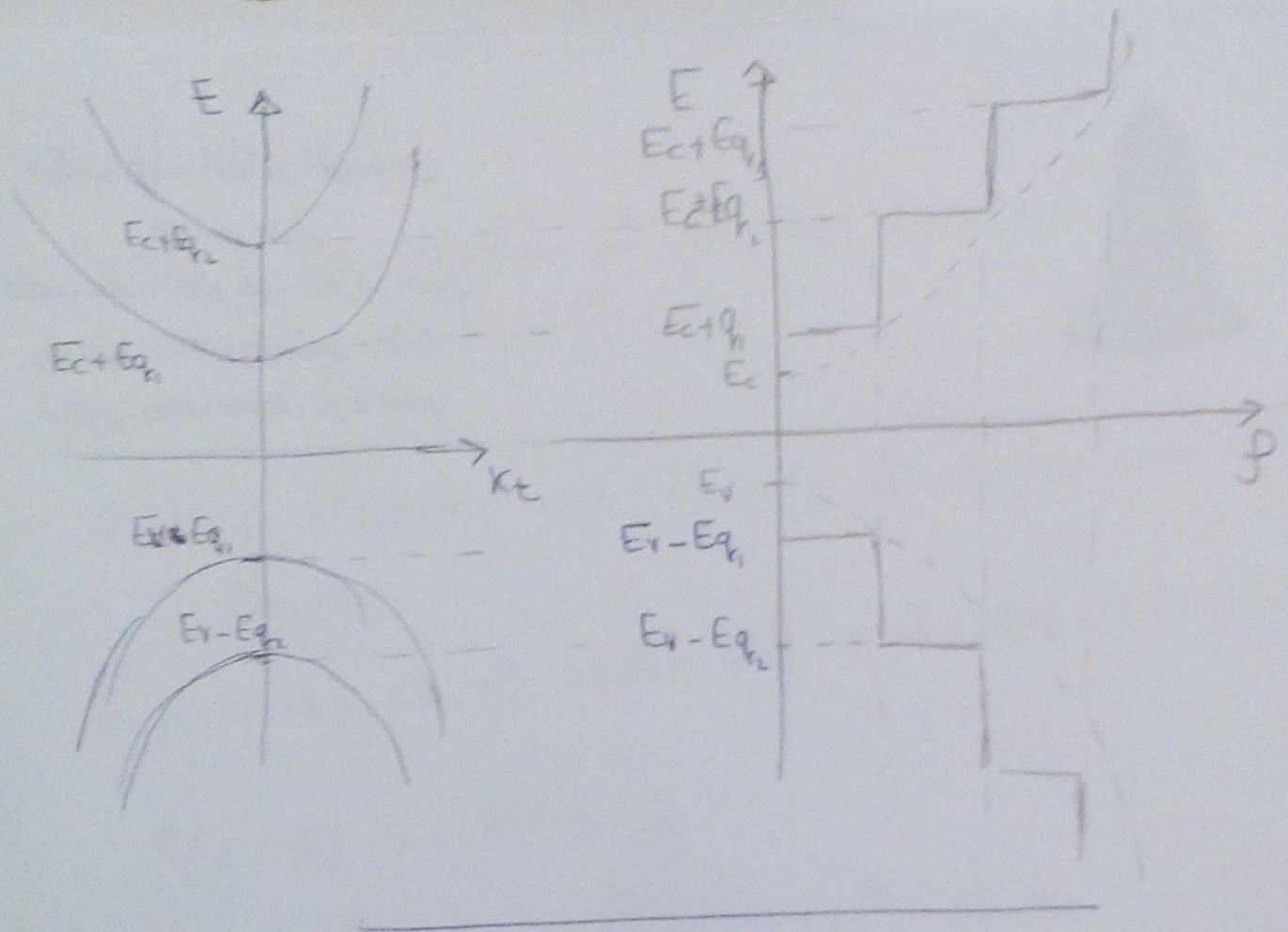
$$E = E_c + \frac{\hbar^2 (k_x^2 + k_y^2 + k_z^2)}{2m_c}$$

$$= E_c + \frac{\hbar^2 k_z^2}{2m_c} + \frac{\hbar^2 (k_x^2 + k_y^2)}{2m_c}$$

$$= E_c + \frac{\hbar^2 \left(\frac{q\pi}{L_z}\right)^2}{2m_c} + \frac{\hbar^2 k_x^2}{2m_c}$$

$$E_1 = E_c + \frac{\hbar^2 \left(\frac{\pi}{L_z}\right)^2}{2m_c} + \frac{\hbar^2 k_x^2}{2m_c} = E_c + E_{q_1} + \frac{\hbar^2 k_x^2}{2m_c}$$





Q(4)

b) Repeat the solution of (a) when dimensions of structure doubles?

~Sol

E-k Diagram and Density of states won't change

only the No. of states will be ~~decreased~~ increased

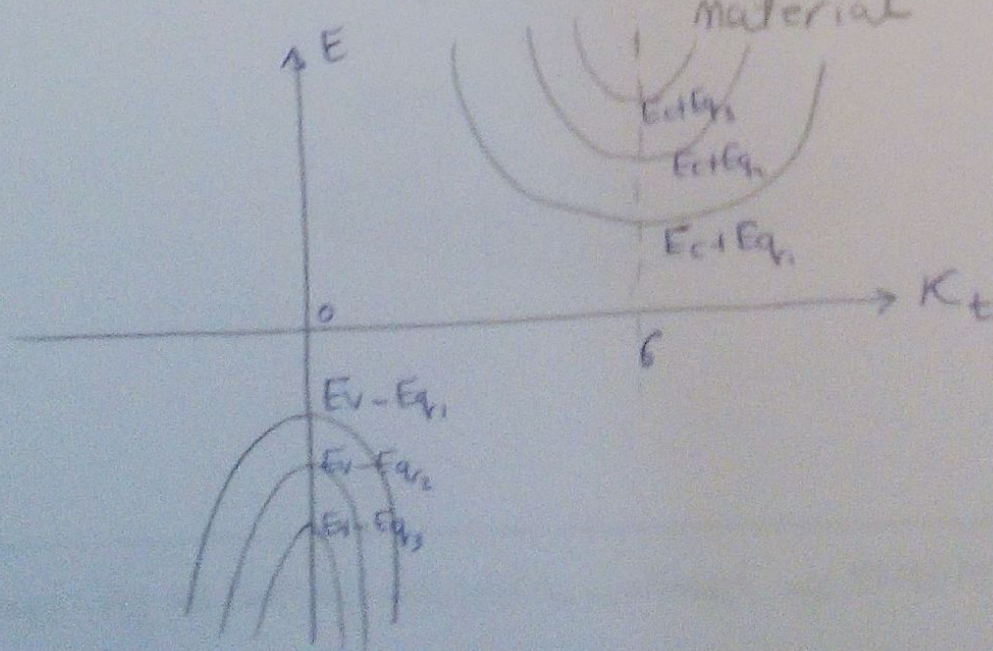
$$N = N \times \text{Volume}$$

$$= 10^{19} \times (0.2)^3 = 8 \times 10^{16} \text{ states}$$

Q(4) c) Repeat the solution of a) when dimensions of structure changed to be

$$1\text{mm} \times 1\text{mm} \times 10\text{\AA}$$

Quantum well not Bulk material



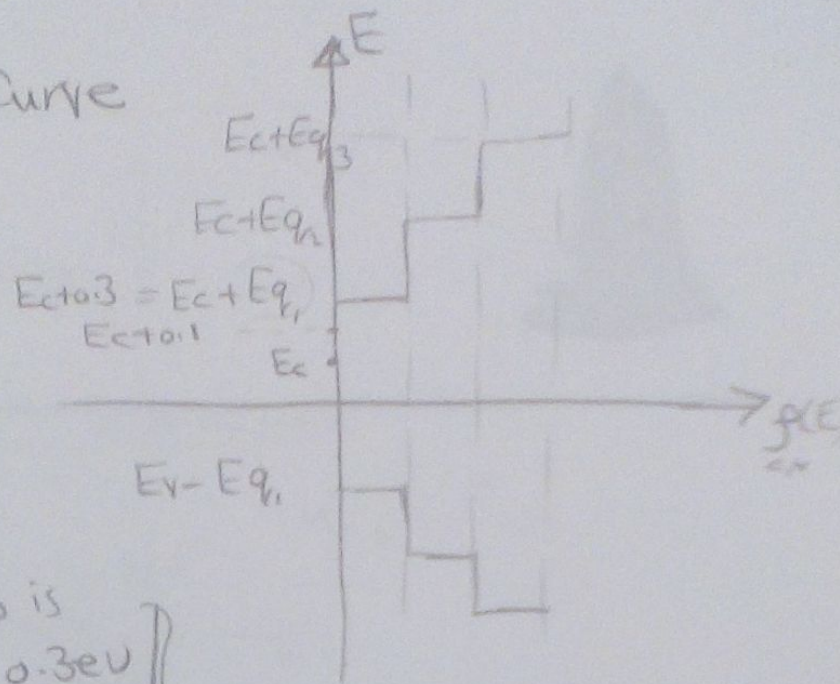
↪ Density of State Curve

$$\rho(E) = \frac{m_c}{\hbar^2 \pi L_z}$$

↪ No. of States from
 $E_c \rightarrow E_c + 0.1 \text{ eV}$

∴ Small energy in C.B is
 $E_c + E_{q_1} = E_c + 0.3 \text{ eV}$

∴ $\rho(E)$ in this region = Zero



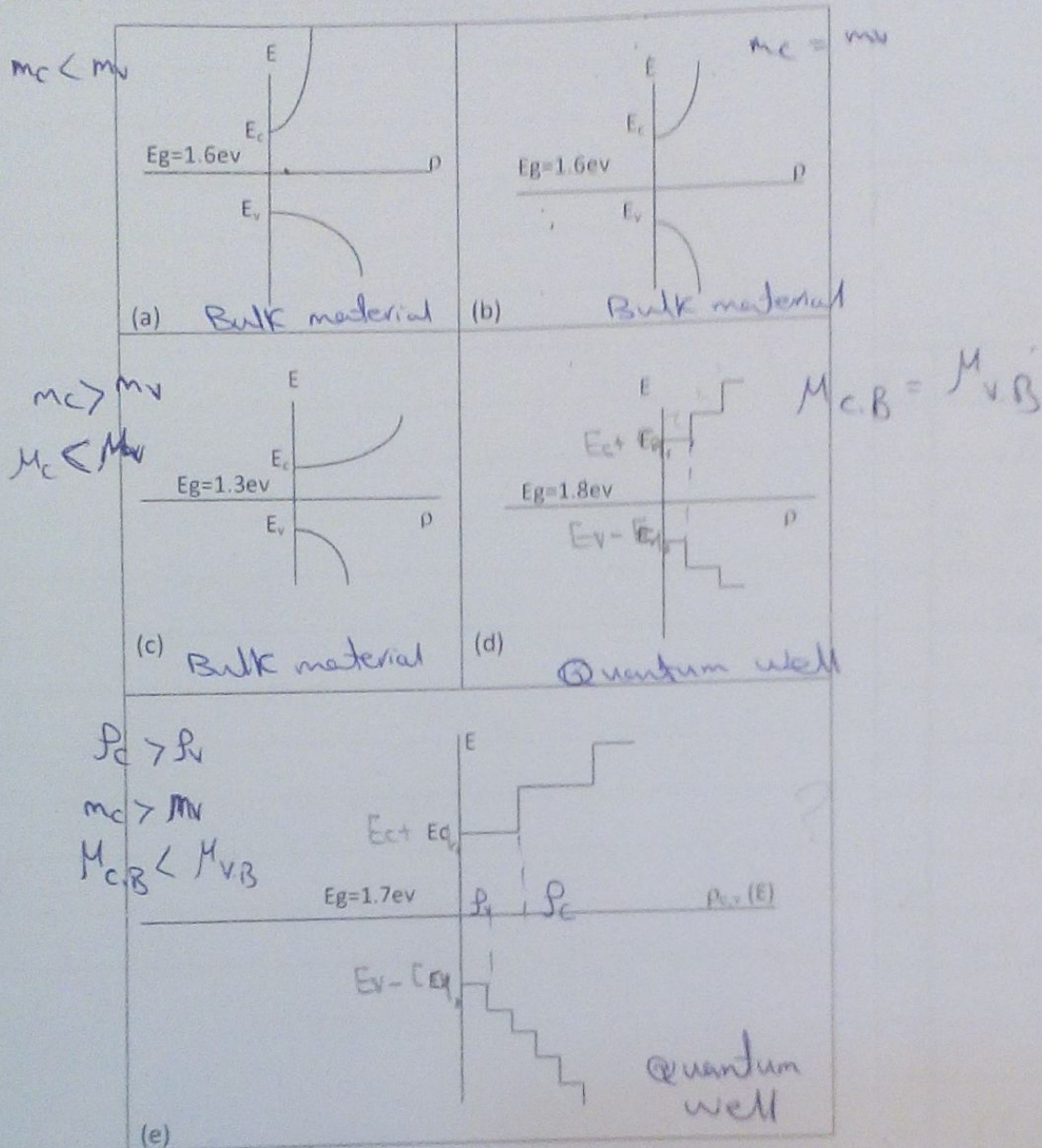
$$E_{q_1} = \frac{\hbar^2 \left(\frac{\pi}{L_z} \right)^2}{2m_c}$$

$$= \frac{(1.05 \times 10^{-34})^2}{2 \times 0.98 \times 9.1 \times 10^{-31} \left(\frac{\pi}{10^{-9}} \right)^2}$$

$$= 6.1 \times 10^{-20} \text{ J}$$

$$E_{q_1} = 0.38 \text{ eV}$$

Q3. Given the following curves for $\rho_{c,v}(E)$



- Determine the type of each structure based on its given density of states.
- Draw E-K diagram qualitatively for each material described by its density state.
- Comment on the carriers ~~mobilities~~ ^{mobilities} in each figure.
- Estimate the values ($E_{m1}, E_{m2}, E_{p1}, E_{p2}, E_{q1}$ and E_{q2}) in curve (d) assuming structure dimensions are $1 \text{ mm} \times 5 \text{ mm} \times 8 \text{ \AA}$ and $m_c = 0.067 m_0$, $m_0 = 9.1 \times 10^{-31} \text{ kg}$.
- Using the information given in (d) calculate the density of states per unit volume at
 - $E = E_c + E_{q1}/2$
 - $E = E_c - E_{q1}/2$
 - $E = E_c + 1.5 E_{q1}$
 - $E = E_c + 2 E_{q1}$
- Using the information given in (d) calculate the number of allowed state within the whole structure for energy ranging from
 - E_c to $E_c + E_{q1}/2$
 - $E_c + E_{q1}$ to $E_c + 2 E_{q1}$
- Estimate the values of allowed states per unit volume for structure represented by curve (a) at energies (a) $E_c - 1 \text{ eV}$ (b) $E_c + 0.1 \text{ eV}$ (c) E_c , assuming $m_c = 0.08 m_0$ and $m_v = 0.28 m_0$.
- Estimate ~~the~~ number of allowed states in the conduction band in structure represented by curve (a) assuming their dimensions are $1 \text{ mm} \times 5 \text{ mm} \times 3 \text{ mm}$. $E_c + 2 \text{ eV}$

Q(3) [d] Estimate the values ($E_{m_1}, E_{m_2}, E_{p_1}, E_{p_2}, E_{q_1}, E_{q_2}$) in Curve (d) assuming Dimensions are

$$1\text{mm} \times 5\text{mm} \times 8\text{\AA}, \quad m_c = 0.067 m_0 \\ m_0 = 9.1 \times 10^{-31} \text{Kg}$$

Sol~

$$1) E_{m_1} = \frac{\hbar^2}{2m_c} \left(\frac{\pi}{L_x} \right)^2 = \frac{(1.05 \times 10^{-34})^2}{2 \times 0.067 \times 9.1 \times 10^{-31}} \times \left(\frac{\pi}{1 \times 10^{-3}} \right)^2$$

$$2) E_{m_2} = \frac{\hbar^2}{2m_c} \left(\frac{2\pi}{L_x} \right)^2 = 4 \times \text{القيمة} =$$

$$3) E_{p_1} = \frac{\hbar^2}{2m_c} \left(\frac{\pi}{L_y} \right)^2 = \checkmark$$

$$4) E_{p_2} = \frac{\hbar^2}{2m_c} \left(\frac{2\pi}{L_y} \right)^2 = 4 \times \checkmark$$

$$5) E_{q_1} = \frac{\hbar^2}{2m_c} \left(\frac{\pi}{L_z} \right)^2$$

$$6) E_{q_2} = \frac{\hbar^2}{2m_c} \left(\frac{2\pi}{L_z} \right)^2 = 4 \times \left(\frac{\hbar^2 \cdot \pi^2}{L_z^2} \right) \cdot \frac{1}{2m}$$

P Using the information given in ① Calculate the no. of allowed state within the whole structure for energy ranging from

i) $E_c \rightarrow E_c + E_{q_1}/2$

ii) $E_c \rightarrow E_c + E_{q_1}$

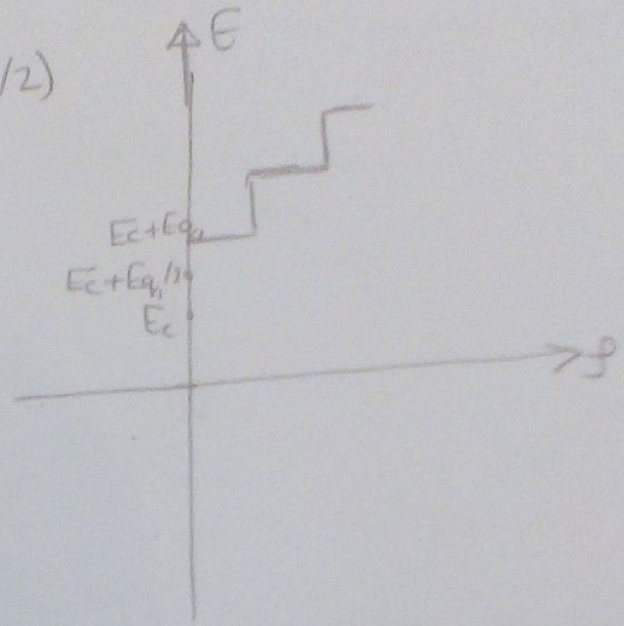
iii) $E_c \rightarrow E_c + 2E_{q_1}$

i) $N \rightarrow (E_c \rightarrow E_c + E_{q_1}/2)$

$N = \text{Zero}$

ii) $E_c \rightarrow E_c + E_{q_1}$

$N = 0$



iii) $N = \int_{E_c}^{E_c + 2E_{q_1}} p(E) dE$
per unit volume

$$= \int_{E_c + E_{q_1}}^{E_c + 2E_{q_1}} p(E) dE = \frac{m_c}{\pi \hbar^2 L_z} [E_c + 2E_{q_1} - E_c - E_{q_1}]$$

$$= \frac{m_c}{\pi \hbar^2 L_z} E_{q_1}$$

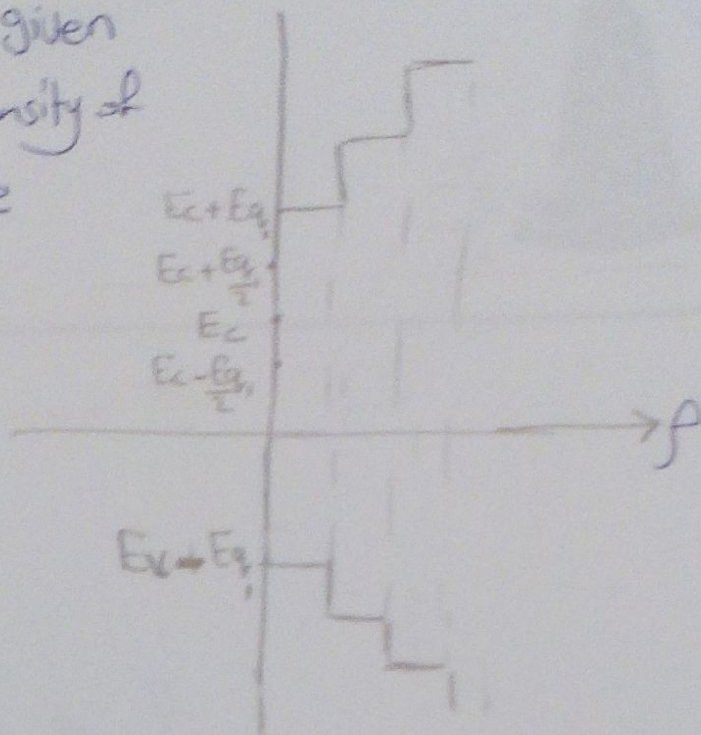
$$= \frac{0.067 \times 9.1 \times 10^{-31} (3.55 \times 10^{-28})}{\pi (1.05 \times 10^{-34})^2 (8 \times 10^{-10})}$$

$$= 7.7 \times 10^{17} \text{ m}^{-3}$$

$$N = 7.7 \times 10^{17} \times (1 \times 10^{-3} \times 5 \times 10^{-3} \times 8 \times 10^{-10})$$

$N = 3.1 \times 10^3 \text{ state}$

e) Using the information given
in [d] Calculate the density of
states per unit Volume
at



i) $E = E_c + E_g/2$

$\hookrightarrow \rho(E) = 0$

ii) $E = E_c - E_g/2$

$\hookrightarrow \rho(E) = 0$

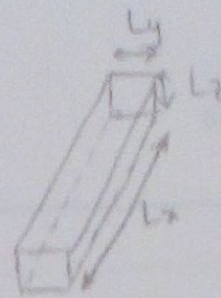
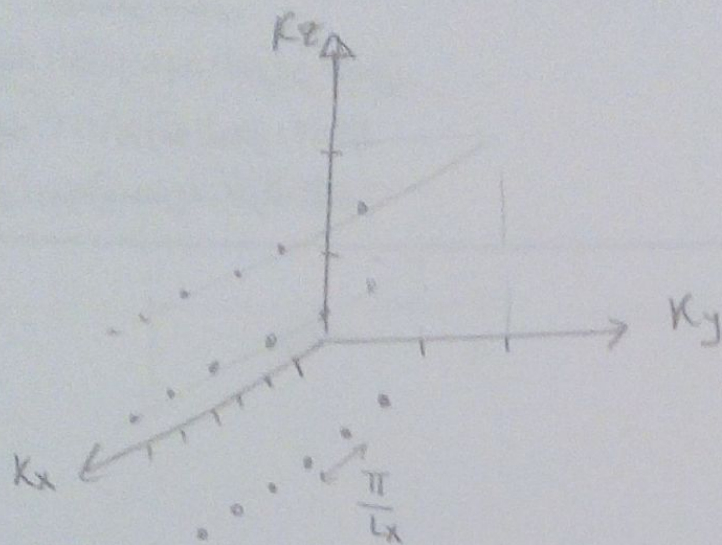
iii) $E = E_c + 1.5 E_g$

$$\begin{aligned} \hookrightarrow \rho_c(E) &= \frac{m_c}{\pi \hbar^2 L_z} = \frac{0.067 + 9.1 \cdot 10^{-31}}{\pi \cdot (8 \cdot 10^{-6}) (1.05 \cdot 10^{-34})^2} \\ &= 2.2 \cdot 10^{45} \text{ m}^{-3} \\ &= 2.2 \cdot 10^{39} \text{ cm}^{-3} \end{aligned}$$

iv) $E = E_c + 2 E_g$

$$\rho_c(E) = \frac{m_c}{\pi \hbar^2 L_z} = 2.2 \cdot 10^{39} \text{ cm}^{-3}$$

□ Density of states $\rho_{c,v}(E)$ for 1D structure (Nano wire)



↳ Length of one state = $\frac{\pi}{L_x}$

↳ In 2D → Area

↳ In 3D → Volume

↳ In 1D → length

↳ length of all allowed state = $K * \frac{1}{2}$
 (الزوجة الموجبة فقط)

↳ $N = 2 * K * \frac{1}{2} = K$
 (Spin of electron)

↳ $N / \text{per unit volume} = \frac{K}{\frac{\pi}{L_x}} = \frac{L_x K}{\pi (L_x L_y L_z)} = \frac{K}{\pi L_y L_z}$

↳ $f(E) = \frac{dN}{dK} \cdot \frac{1}{dE/dK} = \frac{1}{\pi L_y L_z} \cdot \frac{m_e}{\hbar^2 K}$

$$\therefore f(E) = \frac{1}{\pi L_y L_z} \left(\frac{m_e}{\hbar^2} \right) (2)^{-\frac{1}{2}} \left(\frac{m_e}{\hbar^2} \right)^{-\frac{1}{2}} (E - E_c)^{-\frac{1}{2}}$$

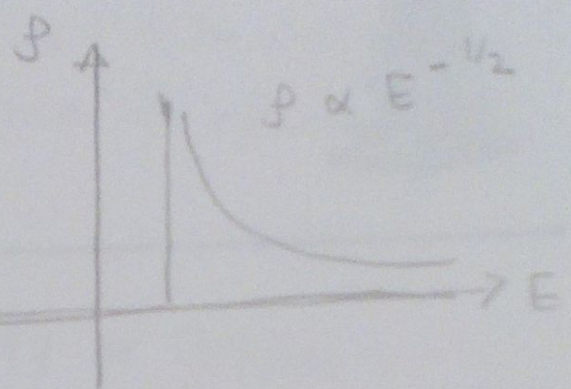
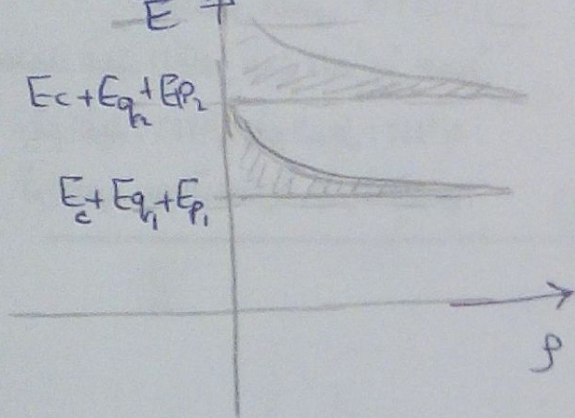
$$= \frac{1}{\sqrt{2} \pi L_y L_z} \left(\frac{m_e}{\hbar^2} \right)^{\frac{1}{2}} (E - E_c)^{-\frac{1}{2}}$$

$$E = E_c + \frac{\hbar^2 K^2}{2m_e}$$

$$\frac{dE}{dK} = \frac{\hbar^2 K}{m_e}$$

$$\therefore K = (E - E_c)^{\frac{1}{2}} \left(\frac{2m_e}{\hbar^2} \right)^{\frac{1}{2}}$$

$$\rho(E) = \frac{\sqrt{mc}}{2} \cdot \frac{1}{\pi \hbar L_y L_z} \cdot (E - E_c)^{-1/2}$$



Mobility

Ability of Carriers (holes/electrons) to move in response of electric field.

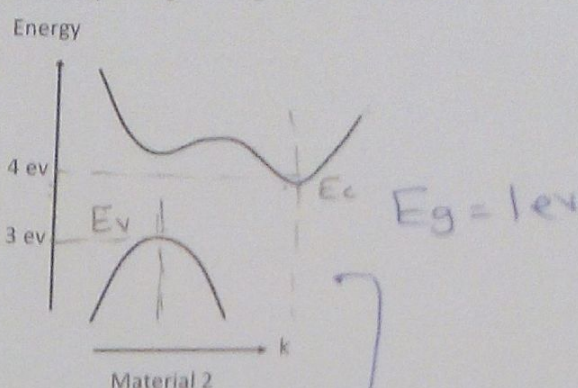
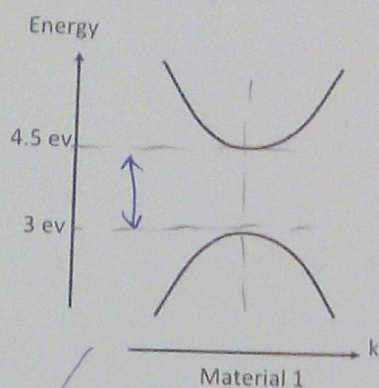
$$\mu = \frac{q}{m^* f_c} \leftarrow \text{Freq. of Collision}$$

$$\mu = \frac{q \tau}{m^*} \leftarrow \begin{array}{l} \text{متوسط الزمن بين} \\ \text{تصادمين متتاليين} \end{array}$$

$$\mu \propto \frac{1}{m^*}$$

From
sheet (1)

Q10. Determine the characteristics (mobility, effective mass, direct or indirect and the energy bandgap) of two semiconductor materials using their corresponding E-k Diagrams shown below.



Direct Band Gap

$$E_g = 1.5 \text{ eV}$$

Effective mass

$$m_c = m_v$$

as Curvature of C.B.
= Curvature of V.B.

Mobility

$$\mu_{C.B.} = \mu_{V.B.}$$

Indirect Band gap

Curvature of C.B. > V.B.
 $m_c < m_v$

$$\mu_c > \mu_v$$

peaks occur at
different (k)

Summary
of Lec (1)

Magento
optic

Q1. Complete the following sentences

- The characteristics of the deflector can be altered by applying Sound
- The characteristics of the Electro optic devices can be altered by applying magnetic fields
- The characteristics of the Electro optic can be altered by applying electric fields
- Optical electronics are defined as devices interact with Light
- Optical circuit board communications are necessary for optical Isolation
- Optical electronic chips are composed of photo detector and Laser diode
- Optoelectronic devices are based on the Semiconductor materials that have electrical conductivities within the range 10^{-6} to 10^3
- The electrical conductivities of the semiconductor materials can be altered by Temperature
Doping and Illumination

Hint: see lecture 1

(18)

↳ Semiconductor Materials

| Semiconductor | Bandgap energy E_g (e.v) |
|--------------------------|----------------------------|
| 1] Silicon Si | 1.1 eV |
| 2] Germanium Ge | 0.67 eV |
| 3] Gallium Arsenide GaAs | 1.42 eV |

Planck - Einstein Relation

∴ Energy of photon = $E = h \cdot f \leftarrow \text{freq.} = h \frac{c}{\lambda}$
 \uparrow
 Planck constant

$$\therefore \lambda = h \cdot \frac{c}{E}$$

↳ To transfer electron from Valence band to Conduction band \rightarrow we need photon with Absorbed

$$E_{ph} > E_g$$

↳ When electron loss its energy and return to Valence band \rightarrow it emit photon

$$E_{ph} > E_g$$

Note

↳ Semiconductor at $T = 0^\circ K \rightarrow$ like insulator
 Valence Band و التوصيل في التوصيل و التوصيل في التوصيل

② at $T = 300^\circ K \rightarrow E = k \cdot T = 0.025 \text{ eV}$

↳ Concentration of Carrier in
 Conduction Band عدد الإلكترونات في التوصيل
 Si $\rightarrow n_i \approx 10^{10} \text{ cm}^{-3}$
 GaAs $\rightarrow n_i \approx 10^6 \text{ cm}^{-3}$
 Ge $\rightarrow n_i \approx 10^{13} \text{ cm}^{-3}$
 عند درجة الحرارة 300K